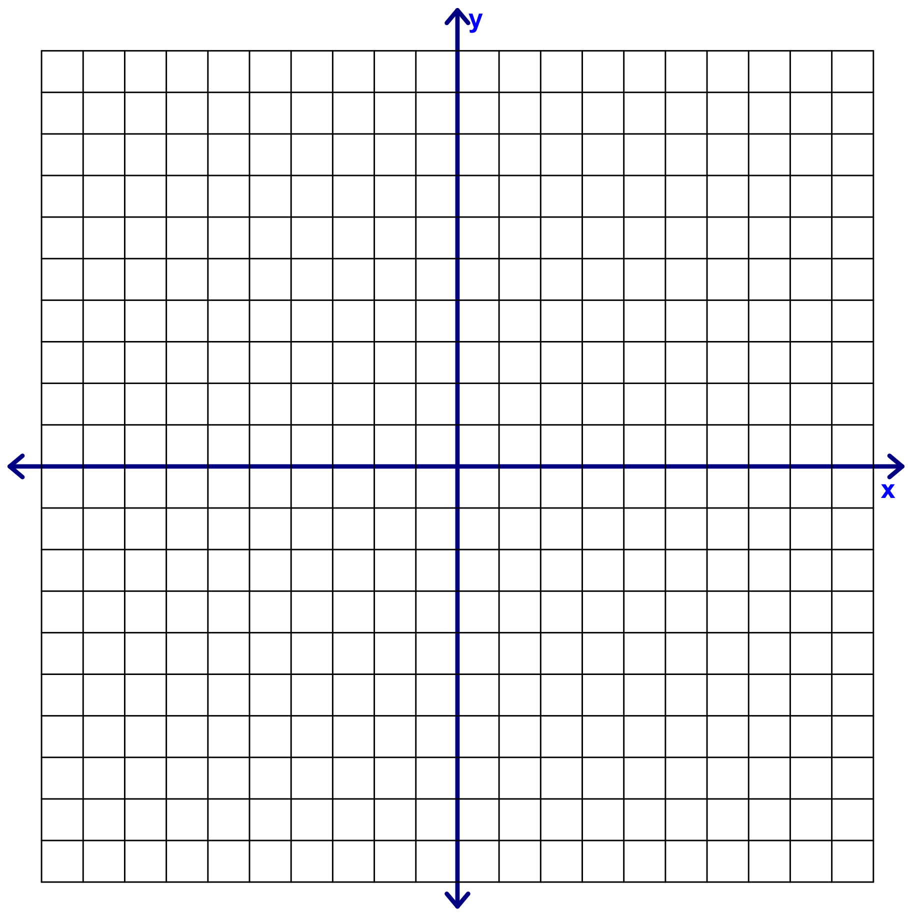
Pre-Calculus CP 1 – Section 3.2 Notes **Name:** ____________________

Logarithmic Functions

Logarithmic Function

The function $f(x) = \log_b x$ is a **logarithmic function** with **base** b, where b is a positive real number and x is any real number.

x	$y = \log_2 x$
-3	
-2	
-1	
½	
0	
1	
2	
3	



⇨ Is there an asymptote anywhere in this graph? Where?

The common base for a logarithm is 10. Let's use this table of values to connect logs to exponentials:

x	-3	-2	-1	0	1	2	3
$y = 10^x$							

Logarithmic Functions

A table of values for $y=10^x$ can be used to solve equations such as $10^x=1000$ and $10^x=\frac{1}{100}$.

However, to solve equations such as $10^x=85$ or $10^x=2.3$, a ***logarithm*** is needed. With logarithms, you can write an exponential equation in an equivalent logarithmic form because they are INVERSES of each other!

Equivalent Exponential and Logarithmic forms

For any positive base b, where $b\neq 1$:

$$y=b^x \quad \leftrightarrow \quad x=\log_b y$$

Ex: Write $5^3=125$ in logarithmic form.

Ex: Write $\log_3 81=4$ in exponential form.

Find the missing parts in the table below:

Exponential form	$2^5=32$		$3^{-2}=\frac{1}{9}$	
Logarithmic form		$\log_{10}1000=3$		$\log_{16}4=\frac{1}{2}$

You can evaluate logarithms with a base of 10 by using the LOG key on a calculator.

Ex: Solve for x: $10^x=85$

Try This: $10^x=\frac{1}{109}$

Logarithmic Functions

Fun Facts:

- The base-10 logarithm is called the **common logarithm**
    - The common logarithm, $\log_{10} x$, is usually written as "$\log x$"

- The base-e logarithm is called the **natural logarithmic function** and is denoted by the special symbol $\ln x$, read as "the natural log of x". Note the natural logarithm is also written without a base. The base is understood to be "e".

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x$ because $a^{\log_a x} = x$
4. If $\log_a x = \log_a y$, then $x = y$ (One-to-One Property)

Ex: Simplify:

a. $\log_4 1$

b. $\log_{\sqrt{7}} \sqrt{7}$

c. $6^{\log_6 20}$

d. $\log_3 x = \log_3 (2x - 4)$

Try This: $\log_2 7x = \log_2 (x^2 + 12)$

Properties of Natural Logarithms

5. $\ln 1 = 0$ because $e^0 = 1$
6. $\ln e = 1$ because $e^1 = e$
7. $\ln e^x = x$ because $e^{\ln x} = x$
8. If $\ln x = \ln y$, then $x = y$ (One-to-One Property)

Use the properties of natural logarithms to simplify each expression:

a. $\ln \frac{1}{e}$

b. $e^{\ln 5}$

c. $\frac{\ln 1}{3}$

d. $2 \ln e$

Application

The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is

$$\beta = 10 \log\left(\frac{I}{10^{-12}}\right)$$

Determine the number of decibels of a sound with an intensity of 2 watts per square meter:

Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter:

Homework: 3.2: p. 236-7 #1, 5, 9, 13, 17, 21, 25, 27-30, 39-44, 45, 49, 53, 57, 61, 65, 79, 85